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# Testability evaluation using prior information of multiple sources

Wang Chao [a,b], Qiu Jing [a,b], Liu Guanjun [a,b,*], Zhang Yong [a,b]

[a] Science and Technology on Integrated Logistics Support Laboratory, National University of Defense Technology, Changsha 410073, China
[b] College of Mechatronic Engineering and Automation, National University of Defense Technology, Changsha 410073, China

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**Abstract** Testability plays an important role in improving the readiness and decreasing the life-cycle cost of equipment. Testability demonstration and evaluation is of significance in measuring such testability indexes as fault detection rate (FDR) and fault isolation rate (FIR), which is useful to the producer in mastering the testability level and improving the testability design, and helpful to the consumer in making purchase decisions. Aiming at the problems with a small sample of testability demonstration test data (TDTD) such as low evaluation confidence and inaccurate result, a testability evaluation method is proposed based on the prior information of multiple sources and Bayes theory. Firstly, the types of prior information are analyzed. The maximum entropy method is applied to the prior information with the mean and interval estimate forms on the testability index to obtain the parameters of prior probability density function (PDF), and the empirical Bayesian method is used to get the parameters for the prior information with a success-fail form. Then, a parametrical data consistency check method is used to check the compatibility between all the sources of prior information and TDTD. For the prior information to pass the check, the prior credibility is calculated. A mixed prior distribution is formed based on the prior PDFs and the corresponding credibility. The Bayesian posterior distribution model is acquired with the mixed prior distribution and TDTD, based on which the point and interval estimates are calculated. Finally, examples of a flying control system are used to verify the proposed method. The results show that the proposed method is feasible and effective.



## 1. Introduction

Testability plays an important role in improving the operational readiness and decreasing the life-cycle cost of equipment.[1,2] Design for testability (DFT) focuses on the fault detection and isolation capability of equipment which are respectively measured by indexes such as the fault detection rate (FDR) and fault isolation rate (FIR). Testability

* Corresponding author. Tel.: +86 731 84573397.
E-mail addresses: zhongwangchao@163.com (C. Wang), gjliu342@qq.com (G. Liu).

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demonstration and evaluation are carried out to check whether the testability indexes meet the requirements in the contract, which is useful to the producer in mastering the testability level and improving the testability design, and helpful to the consumer in making purchase decisions.[3] Testability demonstration is implemented by injecting a certain amount of artificial faults into the equipment and detecting/isolating the faults with predesigned methods. The demonstration result is accepting or rejecting the testability level, which is determined by the total injected fault number and the failed detection/isolation number. Unlike testability demonstration, testability evaluation uses the testability demonstration test data (TDTD) to get the point and interval estimates of the testability indexes.

As the classical testability demonstration method suffers from such problems as a large sample, high risk and a long test period, some testability demonstration planning methods based on small-samples are studied and applied.[4,5] However, the testability evaluation result is not accurate with the traditional evaluation method because of the meager amount of test data. The Bayes method provides theoretical and methodological support for the small sample test as it can take advantage of the prior information. So it is used in the testability index evaluation with small sample test data. Li et al.[6,7] propose a testability evaluation method based on the testability growing test data and field test data, in which the Dirichlet distribution is used as prior probability density function (PDF). The prior data with a growing tendency are used to obtain the parameters of prior pdf, while the field data are used to get the posterior PDF and Markov Chain Monte Carlo (MCMC) method is used to make the evaluation. Chang et al.[8] propose a Bayes testability evaluation model based on prior test data and field test data, in which a hybrid prior PDF is used.

The prior testability information includes the testability prediction information (TPI), the testability expert information (TEI) and testability virtual test data (TVTD). The TPI is acquired by analyzing the testability model with testability modeling and analyzing softwares. The TEI is obtained from the opinions of relevant experts. The TVTD is from the testability virtual prototype (TVP), which is of lower risk and cost, and larger amount of test data.[9–11] However, the prior information listed above cannot be directly used in the testability evaluation without pretreatment for two reasons. First, the forms of the prior information are different, which include the success-fail form TVTD, the prior mean form and the prior interval form under a certain confidence level (TPI and TEI). Therefore, the prior information needs to be transformed to the complete prior pdf. Secondly, the premise of the usage of prior information is data consistency, which means that each source of prior information should be compatible with the TDTD.

As the prior information comes from different sources, a consistency check is needed between each prior information and TDTD.[12] When the consistency check is passed, the similarity between the prior information and TDTD is quantitatively analyzed, That is, the prior credibility of the prior information is obtained, which can be used to balance the weights of the prior information in the evaluation.[12,13]

In the relevant literature on prior information transformation, Li[7] gives a method to convert the TPI and TEI to the success-fail data used in the testability demonstration. Savchuk and Martz[14] propose a reliability evaluation method based on multiple sources of TEI, in which the maximum entropy method is used to transform the prior information of point mean and interval form to beta PDF. The maximum entropy method is also used in Refs.[15–18]. For the prior information with success-fail form, the empirical Bayesian method is usually applied to get the parameters of the beta prior PDF.[19]

The data consistency check methods include the parametrical and the non-parametrical methods. Zhang[12] introduces the procedures of a non-parametrical method under both large and small sample conditions. Tang[20] proposes a parametrical consistency check method of normal distribution under a small sample condition. Liu and Guo[21] use both the parametrical and the non-parametrical methods in the consistency check for success-fail data. On the calculation of prior credibility, based on the method of Ref.[12], Liu and Wu[22,23] give the analytical form of type II error, which makes the calculation of credibility more convenient. Duan and Huang[24] propose a credibility calculation method based on information divergence. Deng and Zha[25] defines the credibility as the enclosed area of the prior and posterior PDFs.

From the analysis above, it can be seen that there is no relevant reference on the testability evaluation using prior information of multiple sources. The commonly used evaluation methods using the prior data of multiple sources in other fields tend to neglect data consistency between prior information and field test data. Furthermore, there are few methods of high operability on data consistency check and prior credibility calculation. So a Bayesian testability index evaluation method based on the prior information of multiple sources and TDTD is proposed, in which data consistency and credibility are fully considered.

The rest of the paper is organized as follows. The overall flow of testability evaluation is introduced in Section 2. In Section 3 the prior PDF parameter calculation methods for different forms are given. In Section 4 the consistency-check method and the prior credibility calculation method are presented. In Section 5 a mixed prior PDF and posterior PDF model is given, based on which the testability index point evaluation and interval evaluation are shown. Section 6 gives examples to verify the proposed method. Section 7 gives the conclusions.

## 2. Flow of testability evaluation

The flow of testability evaluation using prior information of multiple sources is shown in Fig. 1.

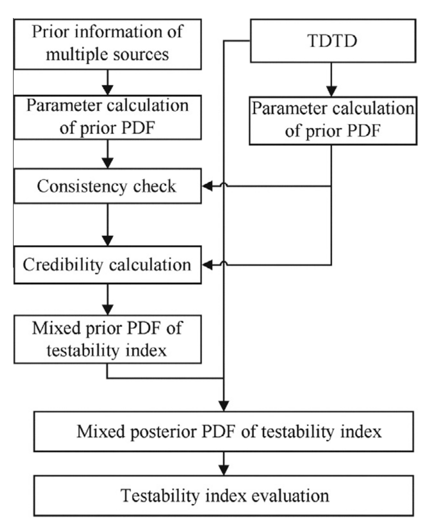


**Fig. 1** Flow of testability evaluation.

Firstly, the prior PDF parameters of prior information and TDTD are calculated. Secondly, consistency checks are made between prior information and TDTD with a parametrical method. For each source of prior information to pass the consistency check, a credibility of the prior information is calculated. Thirdly, a mixed prior PDF of testability index is obtained with every prior PDF and its corresponding credibility. Then the testability index posterior PDF is obtained based on the mixed prior PDF and TDTD. Finally, testability index evaluation is carried out with the posterior PDF.

## 3. Parameter calculation of prior PDF

### 3.1. Parameter calculation for prior information with mean and interval estimate forms

In the testability prior information, the TPI and TEI are given with the two forms:

(1) The point mean $p_0$ of the testability index.
(2) The interval $[p_1, p_2]$ under a confidence level $\mu$.

For the prior information with the point mean form, a point mean $p_0$ is specified for the testability index. Define that $\pi(p)$ is the prior PDF of index $p$. So in terms of Bayes analysis, we have:

$$\int_0^1 p\pi(p)\mathrm{d}p = p_0 \tag{1}$$

For the prior distribution selection with given prior forms, Dyer and Chiou[26] have proved the advantages of beta distribution over other distributions. Furthermore, the test result of testability demonstration test is of a binomial model. So beta distribution or the conjugate distribution of binomial distribution is selected, that is:

$$\pi(p) = \mathrm{beta}(p; a, b) \tag{2}$$

where $a$ and $b$ are the parameters of beta distribution.

The maximum entropy method is used to get the Beta distribution parameters. The Shannon-Jaynes entropy for the prior PDF $\pi(p)$ is[27]:

$$H(\pi) = -\int_0^1 \pi(p)\ln(\pi(p))\mathrm{d}p \tag{3}$$

So the determination of parameters becomes a process to find a combination of $a$ and $b$ which maximize the entropy $H(\pi)$, that is:

$$\begin{aligned} H(\pi) &= H(\mathrm{beta}(p; a, b)) \\ &= -\textstyle\int_0^1 \mathrm{beta}(p; a, b)\ln(\mathrm{beta}(p; a, b))\mathrm{d}p \\ &= \ln(B(a, b)) - a_1\frac{\partial B(a, b)}{\partial a} - b_1\frac{\partial B(a, b)}{\partial b} \end{aligned} \tag{4}$$

where

$$\begin{cases} a_1 = (a-1)/B(a, b) \\ b_1 = (b-1)/B(a, b) \end{cases} \tag{5}$$

$$B(a, b) = \int_0^1 p^{a-1}(1-p)^{b-1}\mathrm{d}p \tag{6}$$

Taking Eq. (2) into Eq. (1), we have:

$$p_0 = a/(a+b) \tag{7}$$

Define that $a^*$ and $b^*$ are the optimal solutions to $a$ and $b$ respectively, and the determination process becomes:

$$H(\mathrm{beta}(p; a^*, b^*)) = \max(H(\mathrm{beta}(p; a, b))) \tag{8}$$

under the restrictions:

$$\begin{cases} a \geqslant 0, b \geqslant 0 \\ bp_0 - a(1-p_0) = 0 \end{cases} \tag{9}$$

In fact, as there exists a proportional relationship between $a$ and $b$, the determination process can be transformed to a single-parameter optimization problem.

For the prior information with the interval form, the index interval under confidence level $\mu$ is $[p_1, p_2]$, so the prior PDF satisfies:

$$\int_{p_1}^{p_2} \pi(p)\mathrm{d}p = \mu \tag{10}$$

The determination of the optimal $a^*$ and $b^*$ solutions becomes:

$$H(\mathrm{beta}(p; a^*, b^*)) = \max(H(\mathrm{beta}(p; a, b))) \tag{11}$$

under the restrictions:

$$\begin{cases} a \geqslant 0, b \geqslant 0 \\ \int_{p_1}^{p_2} p^{a-1}(1-p)^{b-1}\mathrm{d}p - \mu B(a, b) = 0 \end{cases} \tag{12}$$

According to Ref.[27], the entropy of a uniform distribution is 0 and all other distributions have smaller entropy. For the parameter calculation under the restrictions in Eqs. (9), (12), the entropy of beta (1,1) is 0, and for all the other $a$ and $b$ combinations, the entropy is minus. So there is an optimal combination which can both maximize the entropy and satisfy the corresponding restriction.

The solution to Eq. (11) with the restriction of Eq. (12) is difficult. Usually, a gradient method with continuous penalty functions is adopted for its solution. Detailed procedures are shown in Ref.[15,16].

### 3.2. Parameter calculation for prior information with success-fail form

The prior information with a success-fail form, such as the testability trial test data and TVTD, is given as $(n, c)$, where $n$ is the total fault injection times, and $c$ is the failed detection/isolation times in $n$ tests.

Assume that there are $m$ groups of prior data from the same prior success-fail information source, which are defined as $(n_i, c_i)$, $i = 1, 2, \ldots, m-1, m$. The point mean of group $i$ is:

$$\hat{P}_i = \frac{n_i - c_i}{n_i} \tag{13}$$

Empirical Bayes method [19] is used to determine the parameters of the prior PDF as follows.

(1) If $m$ is relatively large, we have

$$\begin{cases} \hat{n} = \dfrac{m^2\left(\sum_{i=1}^m \hat{P}_i - \sum_{i=1}^m \hat{P}_i^2\right)}{m\left(m\sum_{i=1}^m \hat{P}_i^2 - k\sum_{i=1}^m \hat{P}_i\right) - (m-k)\left(\sum_{i=1}^m \hat{P}_i\right)^2} \\ \hat{c} = \hat{n} - \hat{n}\left(\sum_{i=1}^m \hat{P}_i\right)/m \end{cases} \tag{14}$$

where $k = \sum_{i=1}^m n_i^{-1}$.

(2) If $\hat{n}$ derived from Eq. (14) is minus, modify the expression of $\hat{n}$ as

$$\hat{n} = \frac{m-1}{m}\left(\frac{m\sum_{i=1}^{m}\hat{P}_i - \left(\sum_{i=1}^{m}\hat{P}_i\right)^2}{m\sum_{i=1}^{m}\hat{P}_i^2 - \left(\sum_{i=1}^{m}\hat{P}_i\right)^2}\right) - 1 \tag{15}$$

(3) If $m = 1$, Eqs. (14) and (15) are not available. We have

$$\begin{cases} \hat{n} = n \\ \hat{c} = c \end{cases} \tag{16}$$

The parameters of the testability index prior PDF are presented as

$$\begin{cases} a = \hat{n} - \hat{c} \\ b = \hat{c} \end{cases} \tag{17}$$

So the Beta prior PDF is

$$\pi(p) = \text{beta}(p; a, b) = \frac{1}{B(a,b)} p^{a-1}(1-p)^{b-1} \tag{18}$$

## 4. Consistency check and credibility calculation

### 4.1. Consistency check

To ensure the accuracy of the testability evaluation under prior information of multiple sources, we need to make sure that the prior information is of high consistency with TDTD. So data consistency checks are used to assure the compatibilities among all prior information sources and TDTD. However, the prior information and TDTD may not be from the same population. So the consistency check is actually a process to test the comparability of two populations under a certain confidence level.

In Section 3, we have transformed the prior information into complete prior Beta PDFs, so a parametrical consistency check method is adopted. For the prior PDF of each prior information $\pi(p)$, the interval $C = [p_L, p_U]$ with significant level $\alpha$ is determined by:

$$\begin{cases} \int_0^{p_L} \pi(p)\mathrm{d}p = \alpha/2 \\ \int_{p_U}^{1} \pi(p)\mathrm{d}p = \alpha/2 \end{cases} \tag{19}$$

The empirical Bayesian method is used to get the prior PDF $\pi_0(p)$ of TDTD. With a square-error loss function, we can get the point estimate of testability index $\hat{p}$. According to Ref.[21], if $\hat{p}$ satisfies that $p_L \leqslant \hat{p} \leqslant p_U$, the prior information is compatible with TDTD under significant level $\alpha$. If $\hat{p} < p_L$ or $\hat{p} > p_U$, the prior information is not adopted.

### 4.2. Credibility calculation

For the prior information to pass the consistency check, the prior credibility is calculated in this section.

Assume that $X^*$ is the prior information, $X$ is the TDTD. To check the consistency of the two data sources, hypotheses are constructed as

(1) $H_0$: $X^*$ and $X$ are from the same population.
(2) $H_1$: $X^*$ and $X$ are from different populations.

Define that $A$ is the event that accepts $H_0$, $\bar{A}$ is the event that rejects $H_0$.

**Definition.** The credibility of prior information is the probability that the hypothesis $H_0$ is true when accepting $H_0$.

The analytical definition equation of prior credibility is expressed as

$$R = P(H_0|A) \tag{20}$$

With the Bayes theorem, we have

$$R = P(H_0|A) = \frac{P(A|H_0)P(H_0)}{P(A|H_0)P(H_0) + P(A|H_1)P(H_1)} \tag{21}$$

where $P(H_0)$ is the probability of $H_0$ and $P(H_1) = 1 - P(H_0)$.

From the consistency check process, we have

$$P(A|H_0) = 1 - \alpha \tag{22}$$

Define that

$$P(A|H_1) = \beta \tag{23}$$

where $\beta$ is the probability of accepting $H_0$ when $H_1$ is true.

So the prior credibility can be transformed to

$$R = P(H_0|A) = \frac{(1-\alpha)P(H_0)}{(1-\alpha)P(H_0) + \beta P(H_1)} \tag{24}$$

When there is no prior probability for $H_0$ and $H_1$, we consider that the prior probabilities of the two hypotheses are the same, that is, $P(H_0) = P(H_1) = 0.5$. For the TVTD, $P(H_0)$ can be considered as the verification, validation and accreditation (VV&A) result of TVP.

From Eq. (24) we can see that $\beta$ is needed to get the prior credibility. So a parametrical calculation method for $\beta$ is illustrated as the following.

The prior PDF of the prior information is $\pi(p)$, the prior PDF of TDTD is $\pi_0(p)$.

Hypotheses are set as

(1) $H_0$: $\pi(p)$ and $\pi_0(p)$ are the same PDF.
(2) $H_1$: $\pi(p)$ and $\pi_0(p)$ are different PDFs.

In the consistency check, the interval under confidence level $1 - \alpha$ is $C = [p_L, p_U]$. When $H_0$ is true, Eq. (22) is still satisfied. So $\beta$ can be expressed as

$$\beta = P(A|H_1) = \int_C \pi_0(p)\mathrm{d}p \tag{25}$$

Taking $\beta$ into Eq. (24), we can get the prior credibility of prior source $X^*$.

## 5. Testability evaluation model using prior information of multiple sources

### Assumptions

(1) There is prior information from $N$ sources with the forms mentioned above. According to the forms of the prior data, we can get the prior parameters $(a_i, b_i)$, $i = 1, 2, \ldots, N$ for every source.
(2) Consistency checks are implemented between each of the sources and TDTD. Assume that the prior information of $M (M \leqslant N)$ sources pass the check and the prior credibility is $R_j$, $j = 1, 2, \ldots, M$, respectively.

A mixed prior PDF of the testability index is established as

$$\pi(p; a, b) = \sum_{j=1}^{M} w_j \text{beta}(p; a_j, b_j) \tag{26}$$

where $w_j$ is the weight of source $j$ and $w_j$ satisfies that

$$w_j = \frac{R_j}{\sum_{j=1}^{M} R_j} \tag{27}$$

According to the Bayes theorem, the mixed posterior PDF with TDTD $(n, c)$ is expressed as

$$\pi(p|(n,c)) = \frac{1}{C}\sum_{j=1}^{M} w_j p^{a_j+n-c-1}(1-p)^{b_j+c-1}/B(a_j,b_j) \tag{28}$$

where

$$C = \sum_{j=1}^{M} w_j B(a_j+n-c, b_j+c)/B(a_j,b_j) \tag{29}$$

With a square-error loss function $L(p,\hat{p}) = (p-\hat{p})^2$, the posterior point estimate of the testability index $p$ is

$$\hat{p} = \frac{1}{C}\sum_{j=1}^{M} w_j B(a_j+n-c-1, b_j+c-1)/B(a_j,b_j) \tag{30}$$

The variance of the testability index $p$ is

$$\begin{aligned} \mathrm{Var}(p) &= \sigma_p^2 \\ &= \frac{1}{C}\sum_{j=1}^{M} w_j B(a_j+n-c+2, b_j+c)/B(a_j,b_j) - \hat{p}^2 \end{aligned} \tag{31}$$

For the posterior interval estimate under confidence level $1-\gamma$, the limits of the interval can be obtained from:

$$\begin{cases} \int_0^{p_L} \pi(p|(n,c))\mathrm{d}p = \gamma/2 \\ \int_{p_U}^1 \pi(p|(n,c))\mathrm{d}p = \gamma/2 \end{cases} \tag{32}$$

## 6. Examples

The FDR of a flying control system is estimated to verify the proposed method. The system is composed of a power supply, a 1553B bus, an inertial measurement unit, a central computer, an integrated controller and a rudder system. The structure is shown in Fig. 2.

In the prior information, the TPI can be derived from the testability modeling and analyzing software. Here we resort to the testability analysis and design system (TADS) to establish the testability model of the flying control system and make prediction on the FDR.

The TEI is given by the relevant experts who give the mean or interval estimates of FDR according to their design experience and similar testability designs.

The TVTD is derived from the TVP established in Ref.[10]. Virtual fault injection and detection tests are carried out on the TVP and the test data are collected.

### 6.1. Example 1

The prior information of FDR includes

(1) Prediction value of FDR: $p_0 = 0.95$.
(2) TEI 1: with a confidence level 0.95, the interval of FDR is [0.95,0.99].
(3) TEI 2: with a confidence level 0.90, the interval of FDR is [0.90, 0.95].

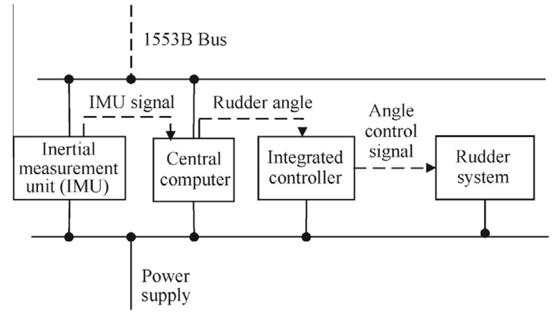


**Fig. 2** Structure of a flying control system.

(4) TVTD: four groups of data with the success-fail form, they are (64, 3), (100, 6), (80, 3) and (124, 8). The VV&A result of the TVP is 0.90.

The TDTD is (43, 2). According to the classical evaluation method, the point estimate and standard deviation of FDR are: $\hat{p}_{\mathrm{FDR}} = 0.9535$, $\sigma_{\mathrm{FDR}} = 0.0321$.

The two-sided interval estimate under 0.90 confidence level is [0.8643, 0.9917] with an interval length of 0.1274.

The two-sided interval estimate under 0.95 confidence level is [0.8419, 0.9943] with an interval length of 0.1524.

According to the methods in Section 2, the parameters of prior PDFs are

(1) TPI: $(a_1, b_1) = (18.278, 0.962)$
(2) TEI 1: $(a_2, b_2) = (296.100, 9.706)$
(3) TEI 2: $(a_3, b_3) = (305.000, 22.540)$
(4) TVTD: $(a_4, b_4) = (307.020, 16.920)$

The significant level of the consistency check is 0.1, the intervals under 0.90 confidence level of each source are

(1) $[p_L, p_U]_1 = [0.8521, 0.9976]$
(2) $[p_L, p_U]_2 = [0.9502, 0.9828]$
(3) $[p_L, p_U]_3 = [0.9068, 0.9526]$
(4) $[p_L, p_U]_4 = [0.9260, 0.9664]$

With a square-error loss function, the point estimate of the TDTD is $\hat{p} = 0.9535$. We can see that the data of Group 3 fail in the consistency check, so the data will not be considered.

For the prior information to pass the check, prior credibility is calculated. For TPI and TEI, the corresponding $P(H_0)$ is set as 0.5. The prior parameters and credibility are shown in Table 1.

According to Eqs. (26) and (28), the prior and posterior pdfs are obtained with the data in Table 1 and the TDTD.

The Bayes point estimate of FDR and posterior standard deviation are: $\hat{p}_{\mathrm{FDR}} = 0.9548$, $\sigma_{\mathrm{FDR}} = 0.0166$.

The two-sided posterior interval estimate of FDR under 0.90 confidence level is [0.9277, 0.9788] with an interval length of 0.0511.

The two-sided posterior interval estimate of FDR under 0.95 confidence level is [0.9199, 0.9822] with an interval length of 0.0623.

Compared with the results of classical evaluation method, the standard deviation of point estimate decreases. The interval lengths under 0.95 and 0.90 confidence levels are smaller

**Table 1** Prior PDF parameters and credibility of the prior information.

| Data source | $a$ | $b$ | Prior credibility $R$ |
|---|---|---|---|
| TPI | 18.278 | 0.962 | 0.4774 |
| TEI | 296.100 | 9.706 | 0.6606 |
| TVTD | 307.020 | 16.620 | 0.9515 |

than the interval lengths of classical method respectively. The introduction of prior information improves the accuracy of the point and interval evaluation.

### 6.2. Example 2

The TPI is obtained by analyzing a testability model without considering the uncertainty in fault detection and isolation and the TEI is usually subjective. The TVTD is from a prototype which is established based on a function-fault-behavior-test-environment model with a VV&A process, so it is more similar to the physical test data.

Based on the analysis above, we take TVTD as a major source of prior information. In the following examples, we will discuss the influences of certain TVTD factors on the evaluation result.

In this example, the TPI and TEI remain the same. Two groups of TVTD are added: (92, 5) and (150, 8) with the same VV&A result of 0.90.

According to the method in Section 2, the prior parameters of TVTD are $(a_4, b_4) = (512.544, 28.549)$ with the added data. The interval estimate under 0.90 confidence level is [0.9306, 0.9620], so TVTD still passes the consistency check and a new credibility can be obtained. The prior parameters and credibility of the prior information are shown in Table 2.

According to Eqs. (26) and (28), the prior and posterior PDFs are obtained with the data in Table 2 and the TDTD.

The Bayes point estimate of FDR and posterior standard deviation are: $\hat{p}_{\mathrm{FDR}} = 0.9543$, $\sigma_{\mathrm{FDR}} = 0.0159$.

The two-sided posterior interval estimate of FDR under 0.90 confidence level is [0.9306, 0.9787] with an interval length of 0.0481.

The two-sided posterior interval estimate of FDR under 0.95 confidence level is [0.9233, 0.9821] with an interval length of 0.0588.

We can see that compared with the estimate results of Example 1, with the same TPI, TEI and VV&A results of TVP, the standard deviation of point estimate decreases and the interval lengths of interval estimate get smaller after increasing the groups of TVTD. The TVTD amount can influence the accuracy of testability evaluation.

To illustrate the differences of the prior and posterior PDFs between Examples 1 and 2, the prior and posterior PDFs of FDR are shown in Fig. 3.

**Table 2** Prior PDF parameters and prior credibility of prior information.

| Data source | $a$ | $b$ | Prior credibility $R$ |
|---|---|---|---|
| TPI | 18.278 | 0.962 | 0.4774 |
| TEI | 296.100 | 9.706 | 0.6606 |
| TVTD | 512.544 | 28.549 | 0.9618 |

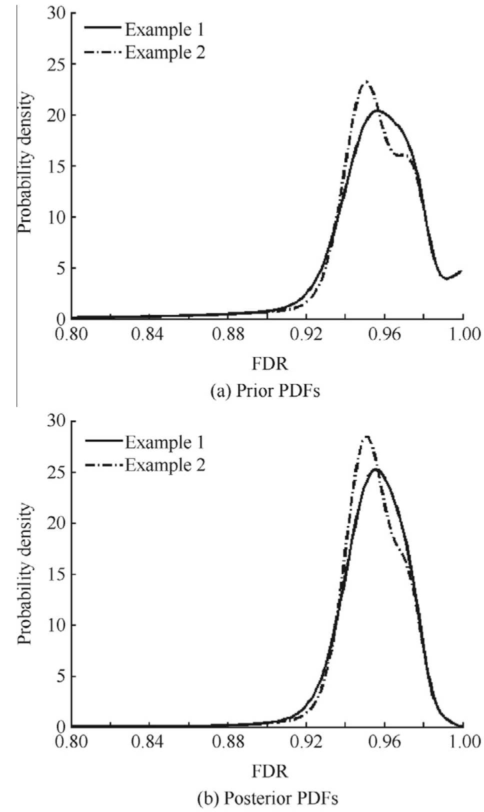


**Fig. 3** Comparison of prior and posterior PDFs used in Examples 1 and 2.

As we can see in Fig. 3(a), compared with the prior PDF of Example 1, the prior PDF of Example 2 has a larger peak value and a narrower shape, which means that the probability density is more concentrated. With the same TDTD, the posterior PDF s are shown in Fig. 3(b). Compared with the posterior PDF of Example 1, the posterior PDF of Example 2 also has a larger peak value and a narrower shape and the probability density is more concentrated. The predominance of the prior and posterior PDFs of Example 2 over the PDFs of Example 1 explains the reason why the increase of TVTD amount can improve point and interval estimate accuracy.

### 6.3. Example 3

In this example, the TPI and the TEI remain unchanged, the amount of TVTD is the same as with Example 2 except that the VV&A result increases to 0.95.

The prior parameters and credibility of the prior information are shown in Table 3. The beta PDFs of FDR are shown in Fig. 4.

The Bayes point estimate of FDR and posterior standard deviation are: $\hat{p}_{\mathrm{FDR}} = 0.9542$, $\sigma_{\mathrm{FDR}} = 0.0158$.

The two-sided posterior interval estimate of FDR under 0.90 confidence level is [0.9307, 0.9786] with an interval length of 0.0479.

The two-sided posterior interval estimate of FDR under 0.95 confidence level is [0.9234, 0.9821] with an interval length of 0.0587.

**Table 3** Prior PDF parameters and prior credibility of the prior information.

| Data source | *a* | *b* | Prior credibility *R* |
|---|---|---|---|
| TPI | 18.278 | 0.962 | 0.4774 |
| TEI | 296.100 | 9.706 | 0.6606 |
| TVTD | 512.544 | 28.549 | 0.9815 |

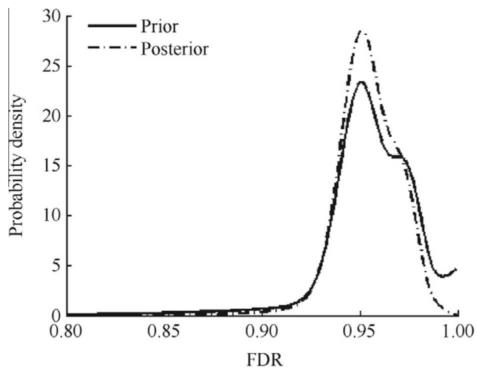


**Fig. 4** Prior and posterior PDFs of FDR.

Compared with Example 2, with the same TPI, TEI and data amount of TVTD, the standard deviation of point estimate decreases and the interval lengths of interval evaluation get smaller after increasing the VV&A result of TVP. The increase in the VV&A result of TVP can modify the accuracy of testability index evaluation.

*6.4. Example 4*

In Example 3, we find that the VV&A result of TVP will influence the accuracy of evaluation. In this example, we will further discuss the influential trend of the VV&A result.

In this example, the TPI and the TEI remain unchanged. The amount of TVTD is the same as with

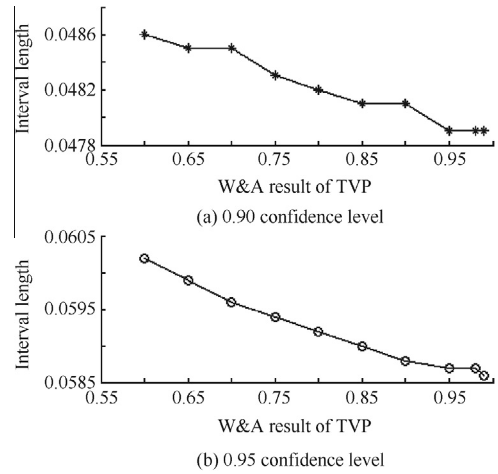


**Fig. 5** Varying tendency of interval lengths under the two confidence levels.

Example 2, but the VV&A result of TVP changes from 0.60 to 0.99, and the corresponding point estimate, interval estimates under 0.90 and 0.95 confidence levels are listed in Table 4.

With the increase of the VV&A result of TVP, the point estimate of FDR increases and the standard deviation decreases. However, when the VV&A reaches a certain level, both the point estimate and standard deviation remain unchanged. For the interval estimate, the FDR intervals under the two confidence levels become more concentrated with decreasing interval lengths. The varying tendency of interval lengths under the two confidence levels are shown in Fig. 5.

As shown in Fig. 5, the interval lengths become smaller with the increase of the VV&A result of TVP. However, we can see that the slopes of the two curves become smaller, which means that the influential ratio on the evaluation result decreases with the increase of the VV&A result.

**Table 4** FDR evaluation under different VV&A results of TVP.

| VV&A result | Point estimate | | Interval estimate | | | |
|---|---|---|---|---|---|---|
| | | | 0.90 confidence level | | 0.95 confidence level | |
| | Point mean | Standard deviation | Interval | Interval length | Interval | Interval length |
| 0.60 | 0.9549 | 0.0162 | [0.9305, 0.9791] | 0.0486 | [0.9224, 0.9826] | 0.0602 |
| 0.65 | 0.9548 | 0.0162 | [0.9305, 0.9790] | 0.0485 | [0.9226, 0.9825] | 0.0599 |
| 0.70 | 0.9547 | 0.0161 | [0.9305, 0.9790] | 0.0485 | [0.9228, 0.9824] | 0.0596 |
| 0.75 | 0.9545 | 0.0160 | [0.9306, 0.9789] | 0.0483 | [0.9229, 0.9823] | 0.0594 |
| 0.80 | 0.9545 | 0.0160 | [0.9306, 0.9788] | 0.0482 | [0.9231, 0.9823] | 0.0592 |
| 0.85 | 0.9544 | 0.0159 | [0.9306, 0.9787] | 0.0481 | [0.9232, 0.9822] | 0.0590 |
| 0.90 | 0.9543 | 0.0159 | [0.9306, 0.9787] | 0.0481 | [0.9233, 0.9821] | 0.0588 |
| 0.95 | 0.9542 | 0.0158 | [0.9307, 0.9786] | 0.0479 | [0.9234, 0.9821] | 0.0587 |
| 0.98 | 0.9542 | 0.0158 | [0.9307, 0.9786] | 0.0479 | [0.9234, 0.9821] | 0.0587 |
| 0.99 | 0.9542 | 0.0158 | [0.9307, 0.9786] | 0.0479 | [0.9234, 0.9820] | 0.0586 |

## 7. Conclusions

(1) A testability evaluation method using prior information of multiple sources is proposed. In the method, a consistency check is used to eliminate the incompatible prior information with the TDTD and the credibility is used to determine the weight of prior information in the mixed prior and posterior pdfs, which makes the evaluation more reasonable. Moreover, the introduction of prior credibility can balance the impact of prior information in the evaluation to prevent prior information of a certain source from overwhelming the TDTD.

(2) The introduction of prior information makes the testability evaluation results much more close to the actual testability level and can modify the accuracy of both point and interval estimates.

(3) The TVTD amount can influence the testability evaluation results. With the increase of the TVTD amount, the probability density of the prior and posterior pdfs become more concentrated, which eventually improves both the point and interval estimate accuracy.

(4) The VV&A result of TVP can also affect the testability evaluation results. Both the point and interval estimate accuracy are improved with the increase of the VV&A result of TVP. However, the influential ratio becomes smaller with the increase of the VV&A result of TVP.

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**Wang Chao** received his B.S. degree in 2007 from Zhengzhou University, Zhengzhou, China, and M.S. degree from National University of Defense Technology (NUDT), Changsha, China, in 2009. He is a Ph.D. candidate at NUDT now. His research interests include testability demonstration and evaluation, Bayes theory and application.

**Liu Guanjun** is a professor and Ph.D. supervisor at the College of Mechatronic Engineering and Automation, NUDT. He received his B.S. and Ph.D. degrees from NUDT in 1994 and 2000, respectively. His research interests include fault diagnosis, testability, maintenance, etc.